



# விடி

KSU GEC BARTON HILL



Reg No.: \_\_\_\_\_

Name: \_\_\_\_\_

**APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY**  
 First Semester B.Tech Degree Examination December 2021 (2019 scheme)

**Course Code: PHT100**

**Course Name: ENGINEERING PHYSICS A**

**(2019-Scheme)**

Max Marks: 100

Duration: 3 Hours

**PART A**

*Answer all questions, each carries 3 marks.*

- 1 ✓ What is Q-factor? How is it related to angular frequency? (3)
- 2 (2) Calculate the frequency of the fundamental note produced by a string 1m long and weighing 2gm kept stretched by a load of 400kg (3)
- 3 ✓ Why does the central fringe of Newton's ring appear dark? (3)
- 4 ✓ What is meant by dispersive power of grating? Give the expression with relevant terms (3)
- 5 ✓ Estimate the uncertainty in the frequency of light emitted by an atom (3)
- 6 ✓ Describe the significance of large surface area to volume ratio of nano materials (3)
- 7 (7) State Gauss' law in magnetism Write the mathematical statement. (3)
- 8 ✓ Define divergence of a vector field Establish its physical significance. (3)
- 9 ✓ What are Cooper pairs? What is their role in superconductivity? (3)
- 10 (10) What is a photo detector? Give two examples. (3)

**PART B**

*Answer one full question from each module, each question carries 14 marks*

**Module-I**

- 11 a) Derive the differential equation of a forced harmonic oscillator and find its solution Define amplitude resonance (10)
- b) A damped oscillator of mass 1 gram has force constant 10 N/M and damping factor  $1 \text{ s}^{-1}$ . Calculate the angular frequency without damping and with damping (4)
- 12 a) Derive an expression for fundamental frequency of transverse vibrations of a stretched string. (10)

- b) A wave is represented by  $y = 3 \times 10^{-3} \cos(8.4 \times 10^{13}t + 2.8 \times 10^5 z)$  where  $y$  and  $z$  are in m and  $t$  in second. Compute the following (4)
- (i) amplitude (ii) frequency (iii) wavelength (iv) wave velocity

## Module-II

- 13 a) Explain the formation of interference fringes in air wedge. How is it used to determine the diameter of a thin wire? (10)
- b) The diameter of the 10<sup>th</sup> and 20<sup>th</sup> Newton's rings formed with a plano-convex lens and an optically plane glass plate are  $0.415 \times 10^{-2}$  m and  $0.616 \times 10^{-2}$  m respectively. If the wavelength of the interfering light is 5893 Å, calculate the radius of curvature of the lens. (4)
- 14 a) Discuss the theory of plane transmission grating and also derive the grating equation. State and explain Rayleigh's criterion for the limit of resolution in the case of grating. (10)
- b) Light of wavelength 656 nm falls normally on a grating 20 mm wide. The first order is 18° from the normal. What is the total number of lines in the grating? (4)

## Module-III

- 15 a) Derive the Schrodinger's time dependent equation for a moving particle and hence derive the Schrodinger's time independent equation. (10)
- b) If an electron's position can be measured to an accuracy of  $2.0 \times 10^{-8}$  m, how accurately can its velocity be known? (4)
- 16 a) Give a brief note on mechanical, optical and electrical properties of nanomaterials. Mention any four applications of nanotechnology. (10)
- b) Explain Quantum confinement. (4)

## Module-IV

- 17 a) Define the terms magnetisation, magnetic flux density, magnetic permeability, relative permeability and susceptibility. Obtain the relation between relative permeability and susceptibility. (10)
- b) Calculate the magnetic flux density and the magnetic moment per unit volume when a magnetising field of  $6 \times 10^5$  A/m applied. Magnetic susceptibility is  $-8.2 \times 10^{-6}$ . (4)
- 18 a) Derive Maxwell's equations from the fundamental laws of electricity and magnetism. (10)
- b) Plane electromagnetic wave (sinusoidal) has maximum intensity of electric field  $200 \times 10^6$  V/m. Calculate  $H_{\max}$ . (4)

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**Module-V**

- 19 a) Describe the phenomenon of superconductivity Define critical temperature and critical magnetic field Mention any four applications of superconductors (8)
- b) Distinguish between Type I and Type II superconductors (6)
- 20 a) Define acceptance angle and numerical aperture of optic fibre Derive an expression for numerical aperture of a step index fibre (10)
- b) The numerical aperture of an optic fibre is 0.5075 and the refractive index of the cladding is 1.475 Calculate the refractive index of the core, acceptance angle, and the critical angle for total internal reflection (4)

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PHT 100 - Engineering Physics A (2019 Scheme)

① Q factor is defined as  $2\pi$  times energy stored in an oscillator to energy loss per period.

$$Q = \frac{2\pi \times \text{Energy stored}}{\text{Energy loss per period}}$$

$$Q = \frac{\omega}{2\pi}$$

It is directly depend upon angular frequency.

②

Linear density,  $m = \frac{2 \times 10^{-3}}{1} = 0.002 \text{ kg/m}$

$$T = 400 \text{ kg} = 400 \times 9.8 \text{ N}$$

$$v = \frac{1}{2l} \sqrt{\frac{T}{m}}$$

$$= \frac{1}{2 \times 1} \sqrt{\frac{400 \times 9.8}{0.002}}$$

$$= \underline{\underline{700 \text{ Hz}}}$$

③

When a beam of monochromatic light is incident normally on Newton's ring apparatus setup, two beams of light are reflected one from top surface of air film and other from bottom surface of glass plate.

They produce an interference pattern. They will be alternate dark and bright band. of increasing radii since the thickness of air film is zero at the centre i.e., it is the point of contact of lens and glass plate.

the central spot will always be dark in a Newton's ring

- ④ In a grating different wavelengths are diffracted through different angles. Dispersive power of a grating is the ratio of change in angle of diffraction to the corresponding change in wavelength

$$\text{Dispersive power, } DP = \frac{d\theta}{d\lambda}$$

- ⑤ An atom remains in excited state for about  $10^8$  s maximum uncertainty in the time can be taken as  $10^8$  s. The corresponding minimum uncertainty in energy is  $\Delta E = \frac{h}{\Delta t} = \frac{h}{10^8}$

$$\text{Since } E = h\nu$$

$$\Delta E = h \Delta \nu$$

$$\Delta \nu = \frac{\Delta E}{h} = \frac{\frac{h}{10^8}}{h} = \frac{1}{10^8} \text{ Hz}$$

$$= 1.67 \times 10^7 \text{ Hz}$$

$$= \underline{\underline{16.7 \text{ MHz}}}$$

This is the minimum uncertainty in frequency measurement.

- ⑥ For spherical nanoparticles the surface to volume ratio is

$$\frac{\text{Surface Area}}{\text{Volume}} = \frac{4\pi R^2}{\frac{4\pi R^3}{3}}$$

$$= \frac{3}{R}$$

$$= \underline{\underline{\frac{3}{R}}}$$

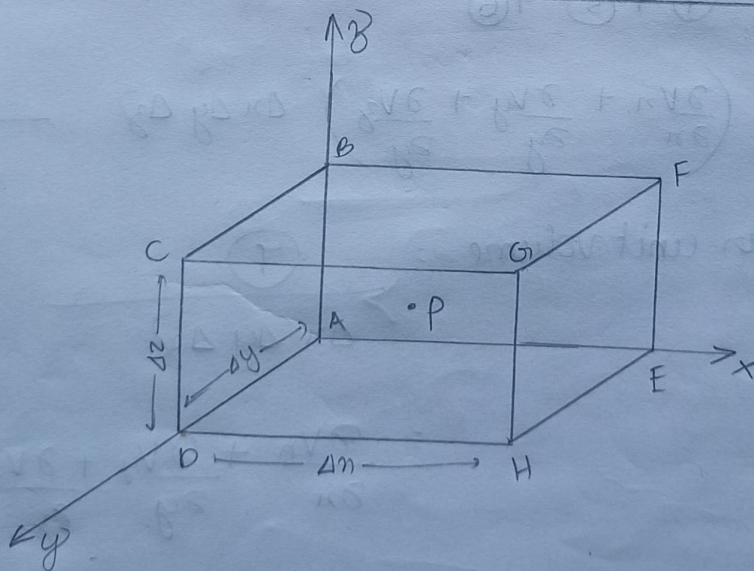
Hence surface to volume ratio increases as  $R$  decreases since it is inversely proportional

For eg: Consider a cube of side  $1\text{ cm}$ . Its total S.A. is  $6\text{ cm}^2$  and volume is  $1\text{ cm}^3$ . If this cube of side  $1\text{ cm}$  is crushed into small pieces of side  $10\mu\text{m}$  we have  $10^9$  such cube. New surface area will be  $10^9 \times 6 \times (10 \times 10^{-6})^2 = 0.6\text{ cm}^2 = 6000\text{ cm}^2$ . If we decrease surface area further to  $10\mu\text{m}$ , there will be  $10^{18}$  such cubes and SA becomes  $600\text{ cm}^2$ .

- (7) Surface integral of normal component of a vector  $\vec{F}$  taken over a closed surface is equal to volume integral of the divergence of vector function over the volume enclosed by the closed surface

$$\iiint_V (\nabla \cdot \vec{F}) dV = \oint_S \vec{F} \cdot d\vec{s}$$

(8)



Consider a fluid in motion. Let  $v(x, y, z)$  be the velocity of fluid at  $p(x, y, z)$

$$\vec{v} = v_x \hat{i} + v_y \hat{j} + v_z \hat{k}$$

$$(\nabla \cdot \vec{v}) = \left( \frac{\partial v_x}{\partial x} \right) + \left( \frac{\partial v_y}{\partial y} \right) + \left( \frac{\partial v_z}{\partial z} \right) \quad \text{--- (1)}$$

x Component of velocity at centre of ABCD =  $V_n - \frac{\partial V_n}{\partial x} \frac{\Delta x}{2}$

x Component of velocity at centre of EFGH =  $V_n + \frac{\partial V_n}{\partial x} \frac{\Delta x}{2}$

Volume of fluid entering through ABCD  $\left\{ \left( V_n - \frac{\partial V_n}{\partial x} \frac{\Delta x}{2} \right) \Delta y \Delta z \right.$  — (2)

Volume of fluid leaving through EFGH  $\left\{ \left( V_n + \frac{\partial V_n}{\partial x} \frac{\Delta x}{2} \right) \Delta y \Delta z \right.$  — (3)

Net outflow through x direction = (3) - (2)

=  $\frac{\partial V_n}{\partial x} (\Delta x \Delta y \Delta z)$  — (4)

Similarly

through y direction =  $\frac{\partial V_y}{\partial y} (\Delta x \Delta y \Delta z)$  — (5)

through z direction =  $\frac{\partial V_z}{\partial z} (\Delta x \Delta y \Delta z)$  — (6)

Total outflow = (4) + (5) + (6)

=  $\left( \frac{\partial V_x}{\partial x} + \frac{\partial V_y}{\partial y} + \frac{\partial V_z}{\partial z} \right) \Delta x \Delta y \Delta z$  — (7)

Net outflow per unit volume = (7)

$\Delta x \Delta y \Delta z$

=  $\frac{\partial V_x}{\partial x} + \frac{\partial V_y}{\partial y} + \frac{\partial V_z}{\partial z}$

(1)



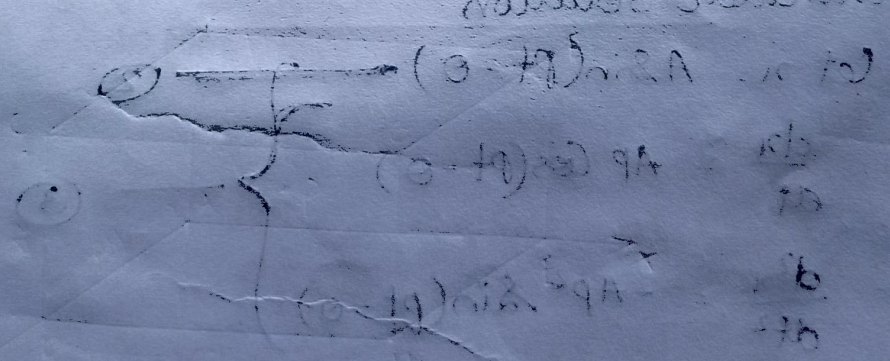
9 Cooper pair is the electron pair formed in a semiconductor at the superconducting state due to electron phonon interaction

Superconducting states is due to cooper pair. At zero kelvin, almost all electrons are coupled as cooper pair.

10 Photodetector is a device used to convert light signals that hit the junction into a voltage or current. Photodetectors are required at receiving end of an optical instrument communication link

eg: photodiode, phototransistor.

$$I_p = \frac{q A \phi_{ph} \eta}{h \nu} \left( \frac{h \nu}{q \phi_{ph}} \right)^{1/2} \left( \frac{q \phi_{ph}}{h \nu} \right)^{1/2} \left( \frac{h \nu}{q \phi_{ph}} \right)^{1/2} \left( \frac{q \phi_{ph}}{h \nu} \right)^{1/2} \dots$$



$$I_p = \frac{q A \phi_{ph} \eta}{h \nu} \left( \frac{h \nu}{q \phi_{ph}} \right)^{1/2} \left( \frac{q \phi_{ph}}{h \nu} \right)^{1/2} \left( \frac{h \nu}{q \phi_{ph}} \right)^{1/2} \left( \frac{q \phi_{ph}}{h \nu} \right)^{1/2} \dots$$

## Part B

### Module I

(11)

(a) In forced harmonic oscillator,

$$F = -kx - bv + F_0 \sin pt$$

$$m \frac{d^2 x}{dt^2} = -kx - b \frac{dx}{dt} + F_0 \sin pt$$

$$\frac{d^2 x}{dt^2} = -\frac{k}{m} x - \frac{b}{m} \frac{dx}{dt} + \frac{F_0}{m} \sin pt$$

$$\boxed{\frac{d^2 x}{dt^2} + 2\gamma x' + \omega_0^2 x = f_0 \sin pt} \quad \text{--- (1)}$$

It is the differential equation of forced harmonic oscillator

Its solution contains 2 parts

(i) Complementary function:  $\frac{d^2 x}{dt^2} + 2\gamma \frac{dx}{dt} + \omega_0^2 x = 0$   
its solution  $x = a e^{-\gamma t} \sin(\omega t + \phi)$

(ii) Particular solution

$$\text{Let } x = A \sin(pt - \theta) \quad \text{--- (2)}$$

$$\frac{dx}{dt} = AP \cos(pt - \theta) \quad \text{--- (3)}$$

$$\frac{d^2 x}{dt^2} = -Ap^2 \sin(pt - \theta)$$

put (3) in (1)

$$-Ap^2 \sin(pt - \theta) + 2\gamma AP \cos(pt - \theta) + \omega_0^2 A \sin(pt - \theta) = f_0 \sin pt$$

$$A(\omega_0^2 - p^2) \sin(pt - \theta) + 2\gamma AP \cos(pt - \theta) = f_0 \sin(pt - \theta) \cos \theta + f_0 \cos(pt - \theta) \sin \theta$$

Equate co-efficients

$$f_0 \cos \theta = A(\omega_0^2 - p^2) \quad (4)$$

$$f_0 \sin \theta = 2\gamma A p \quad (5)$$

Squaring and adding

$$f_0^2 (\cos^2 \theta + \sin^2 \theta) = A^2 (\omega_0^2 - p^2)^2 + 4\gamma^2 A^2 p^2$$

$$f_0^2 = A^2 (\omega_0^2 - p^2)^2 + 4\gamma^2 A^2 p^2$$

$$A = \frac{f_0}{\sqrt{(\omega_0^2 - p^2)^2 + 4\gamma^2 p^2}}$$

ie complete solution is

$$x = a_0 e^{-\gamma t} \sin(\omega t + \phi) + \frac{f_0}{\sqrt{(\omega_0^2 - p^2)^2 + 4\gamma^2 p^2}} \sin(pt - \theta)$$

⇒ Amplitude of the forced harmonic oscillation become maximum at a particular driving frequency close to natural frequency is called Amplitude resonance

(b)  $m = 0.001 \text{ kg}$   
 $k = 10 \text{ N/m}$   
 $\frac{b}{m} = 2\gamma \Rightarrow b = 2m\gamma = 2 \times 0.001 \times 10^3 = 2 \times 10^{-3}$

Without damping

$$\omega = \sqrt{\frac{k}{m}}$$

$$= \sqrt{\frac{10}{10^{-3}}}$$

$$= \underline{100 \text{ rad/sec}}$$

with damping

$$\omega = \sqrt{\frac{k}{m} - \frac{b^2}{4m^2}}$$

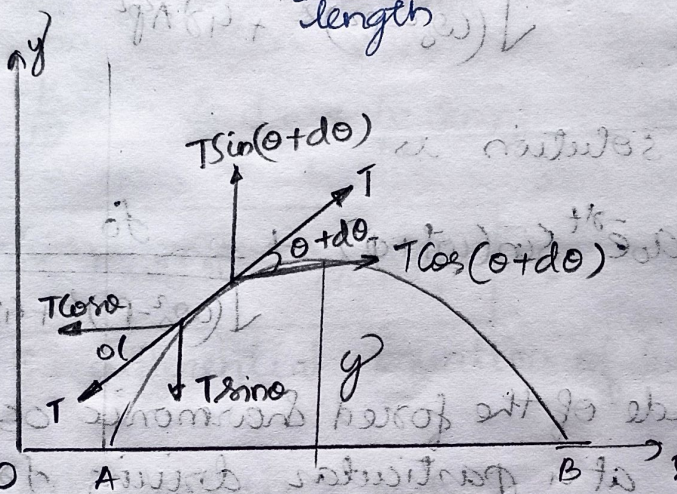
$$= \sqrt{10^4 - \frac{1}{4 \times 10^6}} = \sqrt{10^4 - \frac{10^0}{4}}$$

$$= \sqrt{10^4 - \frac{(2 \times 10^3)^2}{4 \times (10^{-3})^2}} = \sqrt{10^4 - \frac{4}{4}}$$

$$= 99.99 \text{ rad/sec}$$

(12)

(a) Consider a string stretched between A & B under a large tension  $T$ . PQ is an element of length  $dx$  and mass  $mdx$  at a distance  $x$  from the origin, where  $m$  is mass is called linear density length.



Consider  $\theta$  &  $d\theta$  be very small

$$\sum T_x = T \cos(\theta + d\theta) - T \cos \theta = 0$$

$$\sum T_y = T \sin(\theta + d\theta) - T \sin \theta = T d\theta$$

$$\text{slope} = \frac{dy}{dx} = \tan \theta = \theta$$

$$\sum T_y = T d\left(\frac{dy}{dx}\right) = T \frac{d}{dx} \left(\frac{dy}{dx}\right) dx = T \frac{d^2 y}{dx^2} dx$$

$$\text{Force} = T \left(\frac{d^2 y}{dx^2}\right) dx \quad \text{--- (1)}$$

$$\text{Force} = ma = mdx \left(\frac{d^2 y}{dt^2}\right) \quad \text{--- (2)}$$

$$\text{(1) = (2)}$$

$$T \left(\frac{d^2 y}{dx^2}\right) dx = mdx \left(\frac{d^2 y}{dt^2}\right)$$

$\sin \theta \sim \theta$   
 $\cos \theta \sim 1$   
 $\tan \theta \sim \theta$   
 $\sin(\theta + d\theta) \sim \theta + d\theta$   
 $\cos(\theta + d\theta) \sim 1$

$$\frac{d^2y}{dx^2} = \frac{m}{T} \left( \frac{d^2y}{dt^2} \right) \quad \text{--- (3)}$$

$$\frac{d^2y}{dx^2} = \frac{1}{v^2} \left( \frac{d^2y}{dt^2} \right) \quad \text{--- (4) standard eqn in total derivative form}$$

from (3) & (4)

|| label ||

$$\frac{m}{T} = \frac{1}{v^2}$$

$$v = \sqrt{\frac{T}{m}}$$

T = Tension  
m = linear density  
v = velocity of transverse wave

We know

$$v = \frac{\omega}{k}$$

$$\omega = \frac{v}{\lambda}$$

$$v = \frac{1}{\lambda} \sqrt{\frac{T}{m}} \quad \text{As } \lambda = 2l$$

$$v = \frac{1}{2l} \sqrt{\frac{T}{m}}$$

(b)  $y = 3 \times 10^{-3} \cos(8.4 \times 10^{13} t + 2.8 \times 10^5 z)$

(i) Amplitude =  $3 \times 10^{-3} \text{ m}$

(ii)  $\omega = 8.4 \times 10^{13}$

$$\alpha = \frac{\omega}{2\pi} = \frac{8.4 \times 10^{13}}{2\pi}$$

$$= 1.34 \times 10^{13} \text{ Hz}$$

(iii)  $k = 2.8 \times 10^5$

$$\lambda = \frac{2\pi}{k} = \frac{2\pi}{2.8 \times 10^5}$$

$$= 2.24 \times 10^{-5} \text{ m}$$

(wavelength)

(frequency)

(b)

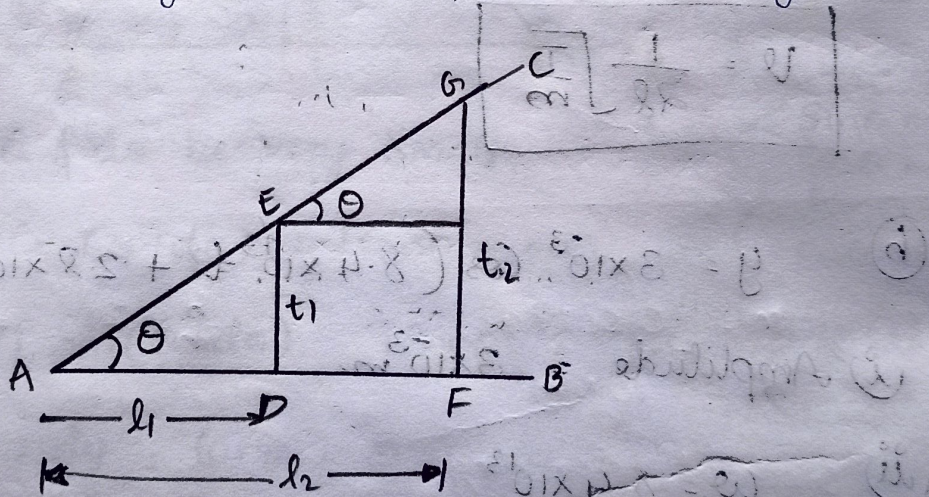
(iv)

$$\begin{aligned}
 v &= \nu \lambda \\
 &= 1.34 \times 10^3 \times 2.24 \times 10^{-5} \\
 &= \underline{3 \times 10^8 \text{ m/s}}
 \end{aligned}$$

Module-II

13

(a) Air wedge consist of two optically plane glass plate placed one over the other in such a way that they are in contact at one end and separated by a small distance at the other end, using a thin metallic wire. A wedge-shaped air film is formed b/w glass plate. When a beam of monochromatic light is incident normally, two beams are reflected and interference pattern is formed. The pattern consist of equidistant, straight parallel, dark and bright band



From figure  $\tan \theta = \frac{t_2 - t_1}{l_2 - l_1}$

Since  $\theta$  is very small,  $l_2 - l_1 = \beta$  is Bandwidth

$$\theta = \frac{t_2 - t_1}{\beta} \quad \text{--- (1)}$$

Condition for darkness of this air film

$$2t \cos r = n\lambda \quad n = 1, 2, 3, \dots$$

$$n = 1 \text{ (air film)}, \quad \cos r = 1 \text{ (normal incidence)}$$

$$2t = n\lambda \quad \text{--- (A)}$$

For  $n^{\text{th}}$  dark band

$$2t_1 = n\lambda \quad \text{--- (2)}$$

For  $(n+1)^{\text{th}}$  dark band --- (3)

$$2t_2 = (n+1)\lambda$$

$$\textcircled{3} - \textcircled{2}$$

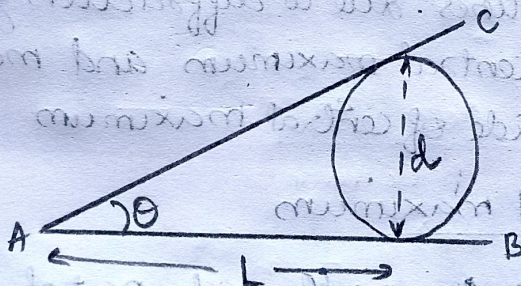
$$2(t_2 - t_1) = (n+1)\lambda - n\lambda$$

$$2(t_2 - t_1) = \lambda$$

$$t_2 - t_1 = \frac{\lambda}{2} \quad \text{--- (4)}$$

put (4) in (1)

$$\boxed{\theta = \frac{\lambda}{2\beta}}$$



Since

$$\theta = \frac{\lambda}{2\beta}$$

From figure  $\theta = \frac{d}{L}$

$$\boxed{d = \frac{\lambda L}{2\beta}}$$

(b)

$$K = 10$$

$$D_{10} = 0.415 \times 10^2 \text{ m}$$

$$D_{10}^2 = 0.172 \times 10^4 \text{ m}$$

$$D_{20} = 0.616 \times 10^2 \text{ m}$$

$$D_{20}^2 = 0.380 \times 10^4 \text{ m}$$

$$\lambda = 5893 \text{ \AA}$$

$$i) \Delta = \frac{D_{20}^2 - D_{10}^2}{4Rk}$$

$$R = \frac{0.380 \times 10^{-4} - 0.172 \times 10^{-4}}{4 \times 10 \times 5893 \times 10^{-10}}$$

$$= 0.88 \mu\text{m}$$

$$= \underline{\underline{88 \text{ nm}}}$$

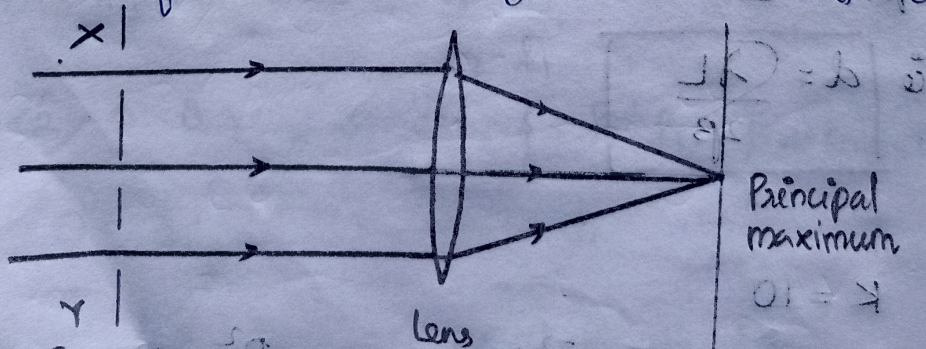
14

(a) Diffraction grating is an arrangement of a large number of narrow rectangular slits of equal width separated by opaque portion.

When monochromatic light is incident on grating, it is transmitted through the slits and is obstructed by lines. Due to diffraction grating effect we can get central maximum and many principle maxima on either side of central maximum.

Condition for central maximum

According to Huygen's principle, each part of wave front incident on grating and passing through the slit send out secondary waves. When focussed using a lens, they will converge to a point at centre of screen and it interfere constructively. This is central maximum.



Grating

lens

Screen

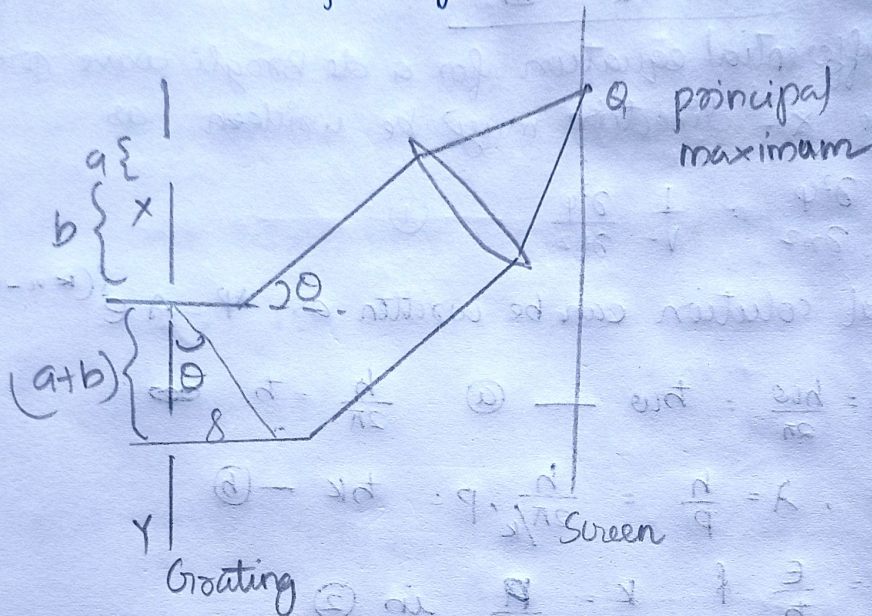
Principal maximum

1



## Condition for Principal maxima

Consider a beam of secondary waves diffracted through an angle  $\theta$  of reaching the point Q such that the path difference is integral multiple of wavelength. This produce principal maxima.



From figure  $\sin \theta = \frac{\delta}{a+b}$  or  $\delta = (a+b) \sin \theta = n\lambda$

i.e.  $\sin \theta = \frac{n\lambda}{(a+b)}$

$(a+b) = \frac{\lambda}{N}$

i.e.  $\boxed{\sin \theta = nN\lambda}$  This is grating equation

\* According to Rayleigh's criteria, two neighbouring spectral lines are just resolved when principal maximum of one wavelength in any order falls on the first minimum of other in the same order.

(b)

$$\sin \theta = nN\lambda$$

$$N = \frac{\sin \theta}{n\lambda}$$

$$= \frac{\sin 18^\circ}{1 \times 656 \times 10^9}$$

$$= 4.7 \times 10^5$$

$$\text{Total number of lines} = \frac{4.7 \times 10^5}{20 \times 10^3} = 235 \times 10^3$$

### Module - III

15

(a) The differential equation for a de Broglie wave propagating along the x-direction may be written as

$$\frac{\partial^2 \psi}{\partial x^2} = -\frac{1}{v^2} \frac{\partial^2 \psi}{\partial t^2} \quad \text{--- (1)}$$

The general solution can be written as  $\psi = A e^{i(kx - \omega t)}$  --- (2)

Energy,

$$E = h\nu = \frac{h\omega}{2\pi} = \hbar\omega \quad \text{--- (a)} \quad \frac{h}{2\pi} = \hbar$$

$$\text{Wavelength, } \lambda = \frac{h}{p} = \frac{h}{2\pi/k} \cdot p = \hbar k \quad \text{--- (b)}$$

$$\text{put } \omega = \frac{E}{\hbar} \quad \& \quad k = \frac{p}{\hbar} \quad \text{in (2)}$$

$$\psi = A e^{i\left(\frac{p}{\hbar}x - \frac{E}{\hbar}t\right)} \quad \text{--- (3)}$$

$$\text{Solution of (1) is } A e^{i\left(\frac{p}{\hbar}x - \frac{E}{\hbar}t\right)} \quad \text{--- (3)}$$

Differentiating (3) w.r.t x twice

$$\frac{\partial \psi}{\partial x} = \frac{\partial}{\partial x} A e^{i\left(\frac{p}{\hbar}x - \frac{E}{\hbar}t\right)} = \frac{i}{\hbar} p \psi$$

$$\frac{\partial}{\partial x} \left( \frac{i}{\hbar} p \psi \right) = \frac{-p^2}{\hbar^2} \psi$$

$$p^2 \psi = -\hbar^2 \frac{\partial^2 \psi}{\partial x^2} \quad \text{--- (4)}$$

Differentiating (3) w.r.t t

$$\frac{\partial \psi}{\partial t} = \frac{\partial}{\partial t} \left( A e^{i\left(\frac{p}{\hbar}x - \frac{E}{\hbar}t\right)} \right) = -\frac{i}{\hbar} E \psi$$

$$\text{ie } E \psi = i \hbar \frac{\partial \psi}{\partial t} \quad \text{--- (5)}$$

$$\text{We know, Total Energy } E = \frac{p^2}{2m} + V$$

$$E \psi = \frac{p^2 \psi}{2m} + V \psi$$

$$i \hbar \frac{\partial \psi}{\partial t} = \frac{\hbar^2}{2m} \frac{\partial^2 \psi}{\partial x^2} + V \psi$$

It is for one dimension

For 3D,

$$i\hbar \frac{\partial \psi}{\partial t} = -\frac{\hbar^2}{2m} \left( \frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} + \frac{\partial^2 \psi}{\partial z^2} \right) + V\psi$$

$\nabla^2$  - Laplacian Operator

$$i\hbar \frac{\partial \psi}{\partial t} = -\frac{\hbar^2}{2m} \nabla^2 \psi + V\psi \quad \text{--- (6) } V \text{ - potential Energy}$$

This is called time dependent Schrodinger wave equation

\* In some cases  $V$  does not depend on time explicitly  
i.e.,  $\psi(x,t) = \phi(x) \cdot f(t)$

$$i\hbar \frac{\partial \psi}{\partial t} = -\frac{\hbar^2}{2m} \nabla^2 \psi + V\psi \quad \text{put } \psi = \phi(x) \cdot f(t) \text{ in (6)}$$

$$\phi(x) \frac{\partial f(t)}{\partial t} i\hbar = -\frac{\hbar^2}{2m} f(t) \nabla^2 \phi(x) + V\phi(x) f(t)$$

$$\div \text{ by } \phi(x) f(t)$$

$$\frac{1}{f(t)} \frac{\partial f(t)}{\partial t} i\hbar = -\frac{1}{\phi(x)} \frac{\hbar^2}{2m} \nabla^2 \phi(x) + V \quad \text{--- (7)}$$

$$\text{LHS} = E$$

$$E = -\frac{1}{\phi(x)} \frac{\hbar^2}{2m} \nabla^2 \phi(x) + V$$

$$\frac{\hbar^2}{2m} \nabla^2 \phi(x) + (E-V)\phi(x) = 0$$

$$i\hbar \left[ \frac{\hbar^2}{2m} \nabla^2 \phi + (E-V)\phi = 0 \right]$$

This is time independent Schrodinger wave equation

$$(b) \quad \Delta p \Delta x = \frac{h}{2\pi}$$

$$m \Delta v \times 2 \times 10^8 = \frac{6.626 \times 10^{-34}}{2\pi}$$

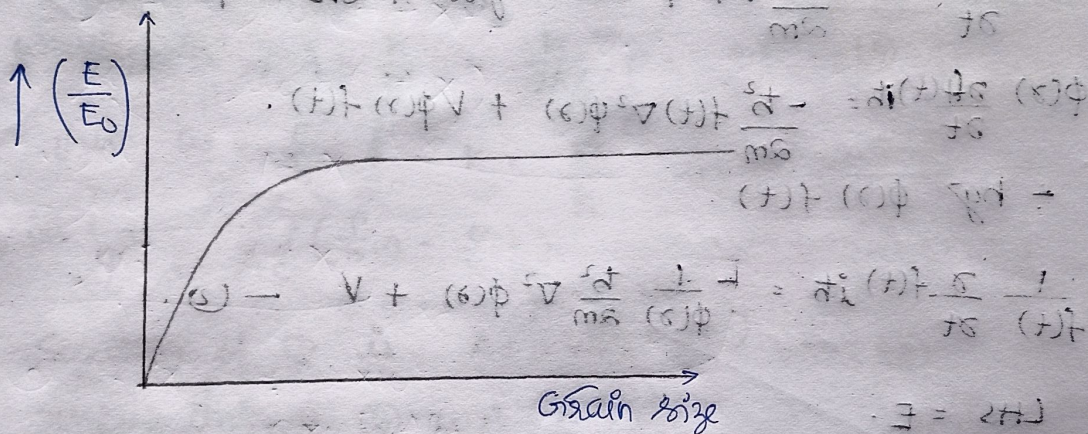
$$\Delta v = \frac{6.626 \times 10^{-34}}{2\pi \times 2 \times 10^8 \times 9.1 \times 10^{-31}}$$

$$\Delta v = \underline{\underline{5794.29 \text{ m/s}}}$$

## ② Mechanical property

The mechanical property of nanomaterial may reach theoretical strength, which are one or two orders of magnitude higher than that of a single crystal in bulk form. The enhancement in mechanical properties is due to reduced probability of defects. In nanomaterials, hardness, young's modulus, yield strength, fracture toughness show significant variation. At nanoscale, strength of metal enhances. For instance, nanocrystalline nickel is as strong as hardened steel. Copper with average grain size 6nm has 5 times higher than grain size of 50 $\mu$ m.

## Electrical Properties



Electrical properties of nanomaterial is due to 4 mechanisms. They are surface scattering, change of electronic structure, quantum transport and effect of microstructure. Electrical conductivity decreases with surface scattering and change in electronic structures. As metal nanowires undergo transition to become semiconductor and semiconductors might become insulators when their diameters are reduced below critical diameter.

## Optical properties

The reduction of material dimension has pronounced effect on its optical properties. This is mainly due to two reasons. One is increased energy level spacing as the system becomes more confined and other is surface plasmon resonance. Surface plasmon resonance is the coherent excitation of free electrons within conduction band upon

interaction with electromagnetic field leading to an inphase oscillation.

### Applications

- \* We can develop new fuel cells using nanotechnology
- \* Can be used for antibacterial treatments
- \* Use in defense field
- \* Nanopowders & coating will increase durability of paint coating.

(b) Quantum confinement is the restricted motion of randomly moving electrons in specific energy level when dimension of material approach the de-Broglie wavelength of electron. Based on dimension that are confined, nanostructures are classified as quantum well (nanosheet), quantum wire, (nano wire) and quantum dots.

### Module - IV

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#### (a) Magnetisation ( $\vec{M}$ )

The strength of magnetic field produced in specimen when placed in an external MF is intensity of magnetisation or magnetisation  $\vec{M}$ .

$$\vec{M} = \frac{\vec{m}}{V}$$

$\vec{m}$  - dipole moment

#### Magnetic flux density ( $\vec{B}$ )

The total number of magnetic field lines passing through a surface normally per unit area is magnetic flux density.

$$\vec{B} = \frac{\Phi}{A}$$

Magnetic permeability ( $\mu$ ): Ease at which a substance allow passage of Magnetic flux through it

$$\mu = \frac{\vec{B}}{\vec{H}}$$

Relative permeability ( $\mu_r$ ): Ratio of absolute magnetic permeability of a medium to that of free space.

$$\mu_r = \frac{\mu}{\mu_0}$$

Susceptibility ( $\chi$ ): It is the ease with which a substance can be magnetised.

$$\chi = \frac{M}{H}$$

We know, Relation b/w susceptibility &  $\mu_r$

$$B = \mu_0(H+M) \quad \& \quad B = \mu H \quad \& \quad M = \chi H$$

$$\mu H = \mu_0(H+M)$$

$$\mu H = \mu_0(H + \chi H)$$

$$\mu H = \mu_0 H(1 + \chi)$$

$$\frac{\mu}{\mu_0} = 1 + \chi$$

$$\boxed{\mu_r = 1 + \chi}$$

(b)  $\chi = -8.2 \times 10^{-6}$

$$H = 6 \times 10^5 \text{ A/m}$$

$$M = \chi H$$

$$= 8.2 \times 6 \times 10^{-6} \times 10^5$$

$$= \underline{\underline{-4.92 \text{ A/m}}}$$

$$B = \mu_0(H+M)$$

$$= 4\pi \times 10^{-7} \times (6 \times 10^5 - 4.92)$$

$$= \underline{\underline{0.75 \text{ T}}}$$

(a) Maxwell's first equation

By Gauss theorem in electrostatics,  $\phi = q/\epsilon_0$

$$\oint \vec{E} \cdot d\vec{s} = \frac{q}{\epsilon_0} = \frac{1}{\epsilon_0} \int \rho dv$$

Apply divergence theorem to LHS

$$\int_V (\nabla \cdot \vec{E}) dv = \frac{1}{\epsilon_0} \int_V \rho dv = \int_V \frac{\rho}{\epsilon_0} dv$$

$$\nabla \cdot \vec{E} = \frac{\rho}{\epsilon_0}$$

We know  $\epsilon_0 \vec{E} = \vec{D}$

$$\nabla \cdot \vec{D} = \rho$$

This is Maxwell's first equation

Maxwell's 2nd equation

By Gauss theorem in Magnetism

$$\phi = 0$$

$$\oint \vec{B} \cdot d\vec{s} = 0$$

$$\nabla \cdot \vec{B} = 0$$

Maxwell's 3rd equation

By Faraday's Electromagnetic Induction

$$\mathcal{E} = - \frac{d\phi_B}{dt} = - \frac{d}{dt} \oint \vec{E} \cdot d\vec{l} = - \frac{d}{dt} \int \vec{B} \cdot d\vec{s}$$

Apply Stokes theorem

$$\int_S (\nabla \times \vec{E}) \cdot d\vec{s} = - \frac{d}{dt} \int_S \vec{B} \cdot d\vec{s}$$

$$= - \int_S \left( \frac{\partial \vec{B}}{\partial t} \right) \cdot d\vec{s}$$

$$\nabla \times \vec{E} = - \frac{\partial \vec{B}}{\partial t}$$

Maxwell's 4th equation

By Ampere's circuital theorem

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 \int_S \vec{J} \cdot d\vec{s}$$

$$\oint \mu_0 \vec{H} \cdot d\vec{l} = \mu_0 \int_S \vec{J} \cdot d\vec{s} \quad \text{Maxwell's 2nd eqn}$$

$$\oint \vec{H} \cdot d\vec{l} = \int_S \vec{J} \cdot d\vec{s}$$

$$\int (\nabla \times \vec{H}) \cdot d\vec{s} = \int_S \vec{J} \cdot d\vec{s}$$

$$\boxed{\nabla \times \vec{H} = \vec{J}} \quad \text{differential form of Ampere's law}$$

Take divergence on both side

$$\nabla \cdot (\nabla \times \vec{H}) = \nabla \cdot \vec{J}$$

$$\text{div. Curl} = 0$$

$$\text{i.e. } \nabla \cdot \vec{J} = 0$$

(only true for steady current)

But for varying current

$$\nabla \cdot \vec{J} = -\frac{\partial \rho}{\partial t}$$

We know

$$\nabla \times \vec{H} = \vec{J} + \vec{J}_d$$

$$\text{But } \vec{J}_d = \frac{\partial \vec{D}}{\partial t}$$

$$\text{i.e. } \boxed{\nabla \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}}$$

(b)

We know that  $\vec{E} = \int \vec{H} \cdot d\vec{l}$

$$\frac{E}{H} = 377 \Omega = \frac{\mu_0}{\epsilon_0} = \frac{4\pi \times 10^{-7}}{8.85 \times 10^{-12}}$$

$$\frac{200 \times 10^6}{H} = 377$$

$$H_{\text{max}} = \frac{200 \times 10^6}{377} = 530 \text{ A/m}$$

$$= 0.53 \times 10^6 \text{ A/m}$$

$$\boxed{\frac{\partial \vec{D}}{\partial t} = \vec{J}_d}$$

Maxwell's 2nd eqn

of Ampere's circuital theorem

$$\oint \vec{H} \cdot d\vec{l} = \int_S \vec{J} \cdot d\vec{s}$$



## Module 1 V

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(a) Super conductivity is the property by which the resistivity of many materials suddenly falls to zero.

\* Critical temperature is the temperature at which a superconductor changes from its normal state to superconducting state.

\* Critical field - The minimum strength of magnetic field required to destroy the superconducting state of a material at a particular temperature ( $T < T_c$ ) is called critical field ( $H_c T$ )

### Applications

- Used to produce very strong and powerful MF
- Used in MRI
- Used as storage device in computer
- In SQUID

### (b) Type 1 Superconductors

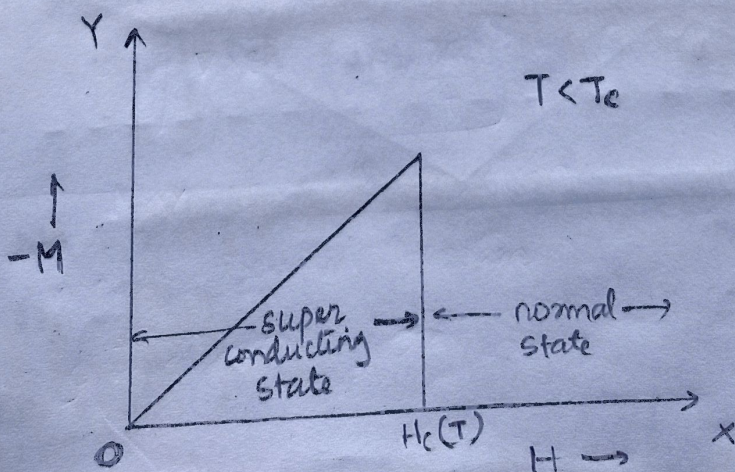
• Super conductor which exhibit very sharp transition from superconducting state to normal state

•  $H_c(T)$  is very small of order 0.1 T

• Superconductivity can be easily destroyed

• Transition at critical field is reversible

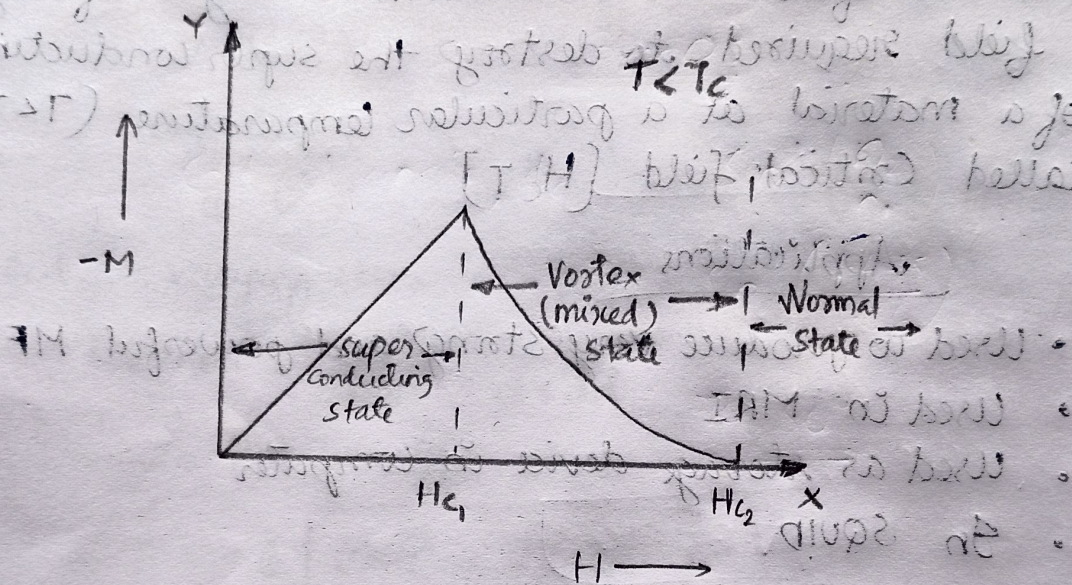
Eg Pb, Sb, Hg, Al, Mg etc.



## Type - II Superconductors

- Exhibit a gradual transition from superconducting state to normal state
- $H_{c2}$  is very high of order 10T to 20T
- Very strong MF is required to destroy superconductivity.
- Transition is irreversible

eg: Nb, Ge, Nb-Sb etc.

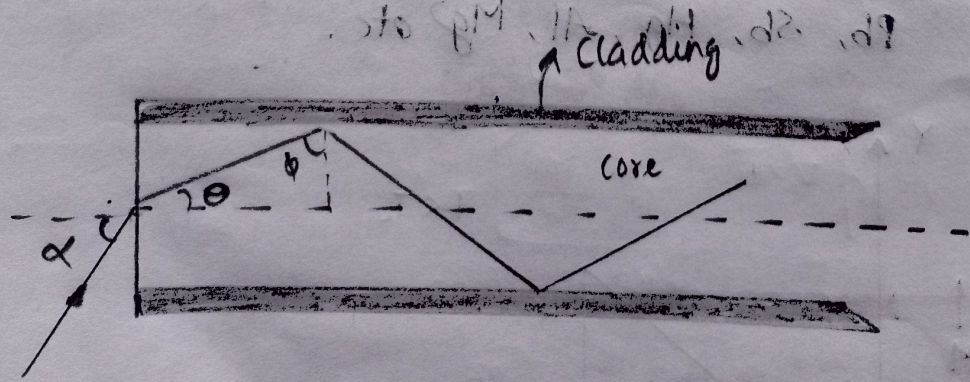


Type I superconductors

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① Acceptance angle is the angle made by incident light at one end of the core with its axis

- Numerical aperture of an optical instrument is a measure of its light gathering capacity and it is defined as sine of maximum value of acceptance angle



Refractive index of core wrt air

$$n_{n_1} = \frac{\sin \alpha}{\sin \theta} \quad \text{--- (1)}$$

$$\therefore n_{n_1} = \frac{n_1}{n_0} = \frac{\sin \alpha}{\sin \theta}$$

$$n_0 \sin \alpha = n_1 \sin \theta \quad \text{--- (1)}$$

From figure  $\theta = \frac{\pi}{2} - \phi$

$$n_0 \sin \alpha = n_1 \cos \phi \quad \text{--- (2)}$$

For critical rays,  $\alpha = \alpha_m$ ,  $\theta = \theta_m$ ,  $\phi = \phi_c$

$$n_0 \sin \alpha_m = n_1 \cos \phi_c \quad \text{--- (3)}$$

Refractive index of core wrt cladding

$$2n_1 = \frac{1}{\sin \phi_c}$$

$$\sin \phi_c = \frac{n_2}{n_1}$$

$$\cos \phi_c = \sqrt{1 - \sin^2 \phi_c}$$

$$\cos \phi_c = \frac{\sqrt{n_1^2 - n_2^2}}{n_1} \quad \text{--- (4)}$$

$$\text{Numerical aperture, } NA = \sin \alpha_m = \frac{\sqrt{n_1^2 - n_2^2}}{n_0}$$

Since  $n_0 = 1$

$$NA = \sqrt{n_1^2 - n_2^2} = \sin \alpha_m$$

(b)  $NA = 0.5075$

$n_2 = 1.475$

$$NA = \sqrt{n_1^2 - n_2^2}$$

$$n_1^2 = NA^2 + n_2^2$$

$$n_1 = \sqrt{NA^2 + n_2^2}$$

Refractive index of core,  $n_1 = \sqrt{NA^2 + n_2^2}$   
 $= \underline{\underline{1.56}}$

①  $NA = \sin \alpha_m$

Acceptance angle  $\alpha = \sin^{-1}(NA)$

②  $\alpha = \sin^{-1}(NA) = \underline{\underline{30.50^\circ}}$

③ Critical angle,  $\phi_c = \sin^{-1}\left(\frac{n_2}{n_1}\right)$

④  $\phi_c = \sin^{-1}\left(\frac{1.475}{1.56}\right)$

$= \underline{\underline{70.80^\circ}}$

$\frac{1}{\sin \phi_c} = \underline{\underline{1.56}}$

$\frac{n_2}{n_1} = \sin \phi_c$

$\sqrt{n_1^2 - n_2^2} = \sin \phi_c$

$\sqrt{n_1^2 - n_2^2} = \sin \phi_c$

⑤

$\frac{n_1^2 - n_2^2}{n_1} = \sin \phi_c = NA$

$L = 0.7$

$NA = \sqrt{n_1^2 - n_2^2} = 0.7$

⑥  $NA = 0.7$

$NA = 0.7$